The goal of this annotated bibliography is to identify, summarize, and categorize resources that address how instruction aligned with the Common Core State Standards (CCSS) in mathematics can be used to provide access to rigorous mathematical learning and further language development for English learners (ELs). The intended audience for this bibliography is K-12 classroom teachers, district instructional leaders and coaches, and administrators. Over 125 articles and papers were reviewed and 30 sources were selected for inclusion. Priority was given to recent articles in peer-reviewed academic and professional publications that would be of practical interest to teachers and administrators in K-12 education. The resources are organized into five main categories with different audiences in mind.

Category 1: Common Core State Standards Mathematics Demands. This category includes articles that address the challenges and opportunities in the Common Core mathematics standards and the changes called for in instruction for English learners (ELs) and all students.

Category 2: Systemic (District, School-wide) Support for Instruction for ELs. This category is geared for classroom teachers, coaches, and administrators who guide and support content area (including mathematics) instruction. This category includes sources that provide districts and schools with resources and ideas for systems to support all teachers and students, with particular attention to the needs of ELs.

Category 3: Discipline-Specific Academic Language Demands of the Mathematics Register and Implications for Instruction. This category includes articles that discuss the particular academic language (speaking, listening, reading, and writing) demands of mathematics that teachers must emphasize in their instruction in order to scaffold and support unfamiliar or complex language to provide access to rigorous content for all students, and English learners in particular.

Category 4: Effective and Culturally Relevant Mathematics Instruction for English Learners. This category includes articles that address a variety of topics related to providing effective and culturally relevant mathematics instruction. Any of the articles could be used as a resource for professional learning and are targeted toward teachers who want to learn more about and improve aspects of their teaching for diverse students. Topics covered include: culturally responsive mathematics teaching, scaffolding mathematics instruction, scaffolding student mathematical discourse, and lesson planning.

Category 5: The Role of Curriculum, Instruction, Professional Learning, Collaboration, and Coaching. This category is intended for instructional leaders and administrators who are
charged with making decisions about curricula to adopt, professional learning opportunities for teachers, and instructional strategies to prioritize and support. The articles included in this section provide guidance and support for teacher professional learning in mathematics instruction.

Following the annotated bibliography is a list of other resources of interest to educators who want to implement an integrated approach to culturally relevant mathematics, language, and literacy instruction for ELs. The first group are all Internet-based sources and include: 1) websites for integrated models of curriculum and professional development programs available to districts; 2) free online tools for teachers for integrating mathematics and language instruction; and 3) useful sites for additional resources for professional learning about mathematics instruction for ELs. The second group of sources includes books and manuals for practitioners on effective mathematics instruction for ELs.

Following is an outline of the contents of this packet and a chart listing the annotated articles alphabetically.

The Literature

1. Common Core State Standards Based Mathematics Instruction
2. Systemic (District, School-wide) Support for Mathematics Instruction for ELs
3. Discipline-Specific Academic Language Demands of the Mathematics Register and Implications for Instruction for ELs
4. Effective, Culturally Relevant Mathematics Instruction for ELs
5. The Role of Instruction, Professional Learning, Teacher Collaboration, and Coaching
   a. Teacher Professional Learning in Credential Programs
   b. Teacher Professional Learning While Employed by a School or District
   c. Teacher Collaboration, Lesson Planning and Data Analysis

Internet Resources

1. Math curricula and instructional models
2. Free tools for teachers
3. Resources for professional learning
**Books and Manuals**

**Table of Articles**

Below is an alphabetical list of articles included in the annotated bibliography. Checkmarks in the columns to the right indicate the topics addressed in each article.

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<td>Office of English Language Acquisition (OELA at the U.S. Department of Education.) English Learner Toolkit. U.S. Department of Education.</td>
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The Literature

A. Common Core State Standards Based Mathematics Instruction

https://www.ets.org/Media/Research/pdf/seeing_the_future.pdf

This report provides a useful overview for a broad audience on the topic of the adoption of common standards as a historic shift for American education. The report describes the important changes in classroom instruction that must take place in order for all students to have full opportunity for college and career readiness. Fourteen content experts from Maine to California in elementary through college review sample test items and tasks and discuss their implications for classroom instruction. Specific examples in English Language Arts and Mathematics are provided for elementary, middle school, and high school levels. The report ends with recommendations for how parents can support their children working on CCSS.

https://csmp.ucop.edu/resources/materials/pl_feb12_goldenberg.pdf

This article states that the Common Core State Standards offer little specific acknowledgement of the challenges English learners will face in attaining college and career readiness. The CCSS website (www.corestandards.org) gives general guidelines for applying the standards to ELs, and recommends schools provide:

1. Appropriate instructional supports to make grade-level course work comprehensible
2. Modified assessments that allow ELs to demonstrate their content knowledge
3. Additional time for ELs to complete tasks and assessments
4. Opportunities for classroom interaction (both listening and speaking) that develop concepts and academic language in the disciplines
5. Opportunities for ELs to interact with proficient English speakers
6. Opportunities for ELs to build on their strengths, prior experiences, and background knowledge
7. Qualified teachers who use practices found to be effective in improving student achievement

The authors acknowledge that teachers are faced with both teaching ELs content and helping ELs develop language proficiency, particularly the academic English in oral and written language that is foundational to the standards. The following guidelines are provided for instruction in the content areas:

- **Recognize the challenge** that English learners face as they attempt to master content and the language in which content instruction is delivered simultaneously
- **Understand that effective teaching** for English learners is similar in many ways to effective teaching for native English speakers. ELs also need focused development of oral reading fluency, vocabulary, reading comprehension, and writing.
• **Utilize sheltered instruction** to make academic content accessible to English learners and include modifications such as: targeting both languages in content objectives; making expectations clear, focused and systematic; employing visuals and charts; using primary language support; choosing reading material with familiar content; and providing additional practice.

• **Evaluate effectiveness** of strategies utilized to support English learners

• **Focus on uses of academic language** as a vital part of content-area instruction. Teachers must have knowledge of the academic language demands needed to learn about, discuss, and write about content area learning.

The following guidelines are provided for promoting English language proficiency:

• **(Designated) English language development (ELD)** must be provided for all ELs. The focus of ELD and content area lessons should complement and reinforce each other.

• **Daily language instruction** should include explicit teaching of elements of English, conversational conventions, and strategies for how to learn the language

• **Academic language** must be taught during ELD instruction and connected to what students are learning in their content area classes so what they are taught can be applied to academic tasks

• **Structured student talk** results from open-ended prompts or prompts for a specific language form or vocabulary item

• **Sufficient duration of services** ensures that ELs participate in ELD at least until reaching advanced (or bridging) language proficiency

• **Grouping** is effective and tailored to meet ELs’ language needs

• **Structured verbal interactions** provide ELs preparation and practice to participate in productive verbal exchanges


This report describes both the challenges districts face in implementing the math standards and best practices for addressing these challenges. Specific examples include ways to create a culture of high expectations for all students, models for supporting teachers as they design and implement rigorous curriculum, and strategies for meaningfully engaging students and parents with the standards. In highlighting practices from the field, the report illuminates how districts can ensure the standards not only raise the bar for all students but also help close achievement gaps for students of color and low-income students. The report includes ten equity-related questions every district and school should be able to answer about common core math.
B. Systemic (District, School-wide) Support for Mathematics Instruction for ELs

* Office of English Language Acquisition (OELA at the U.S. Department of Education.) English Learner Toolkit. U.S. Department of Education.
https://www2.ed.gov/about/offices/list/oela/english-learner-toolkit/eltoolkit.pdf

This toolkit was designed to help state and local education agencies (SEAs and LEAs) in meeting their legal obligations under federal law to ensure that English learners (ELs) have equal access to high quality education and the opportunity to achieve their full academic potential. The toolkit was updated in November 2016 to align with legal requirements of ESSA. Each of the ten chapters provides: (1) explanations of the civil rights and other legal obligations to ELs; (2) checklists SEAs, LEAs and schools can use as self-monitoring tools; (3) sample tools that may be used or adapted to aid with compliance; and (4) additional resources that may provide further relevant information and assistance. The chapters include tools and resources for identifying all ELs, providing ELs with a language assistance program, staffing and supporting an EL program, providing ELs meaningful access to core curricular and extracurricular programs, creating an inclusive environment for and avoiding unnecessary segregation of ELs, addressing ELs with disabilities, serving ELs who opt out of EL programs, monitoring and exiting ELs from EL programs and services, evaluating the effectiveness of a district’s EL program, and ensuring meaningful communication with parents.


This report was developed in response to reforms made to the Elementary and Secondary Education Act (ESEA) that waived certain provisions of the law in exchange for reforms by states related to four principles: (1) achieving college- and career-ready expectations for all students; (2) developing differentiated recognition, accountability, and support systems; (3) supporting effective instruction and leadership; and (4) reducing duplication and unnecessary burden. This report focuses on the implementation of reforms related to ELs across the first three principles. The recommendations are based on the review of 34 approved flexibility plans and describe the requirements for each principle, discuss how ELs were addressed in the approved plans, and provide considerations, based on knowledge of research, for the implementation of proposed reforms. Recommendations for college- and career-readiness and the challenges for ELs include:

- Build capacity to develop ELs’ academic language in content area classrooms
  - Provide professional development and standards-based curricula and instructional materials
  - Support collaboration between language and content teachers
  - Establish mechanisms for sharing effective practices across classrooms, schools, and districts
- Provide ELs with multiple pathways for acquiring grade-level content knowledge and skills
- Develop valid and reliable content-area assessments for ELs
- Ensure that content-area teachers are well prepared and collaborate to support the success of ELs
The report provides “researcher reviewed and endorsed” ideas for supporting the needs of both ELs and the educators who teach them. It also includes additional resources for each principle.

C. Discipline-Specific Academic Language Demands of the Mathematics Register and Implications for Instruction


The author explores perspectives on bilingual mathematics learners and considers how a situated and sociocultural perspective can inform the work in this area. The author acknowledges that research on mathematics instruction for English learners has emphasized vocabulary and comprehension skills in the past. More recent research on mathematics learning has focused on how students construct knowledge, negotiate meanings, and participate in mathematical communication. The current standards emphasize communication and students are now expected to communicate mathematically, both orally and in writing, and participate in mathematical practices, such as explaining solutions, describing conjectures, proving conclusions, and presenting arguments. The author discusses the importance of acquisition of vocabulary but cautions that it is not sufficient for building students’ abilities to fully communicate mathematically. The author defines the “mathematic register” as “the meanings that belong to the language of mathematics.” The author proposes that a situated-sociocultural perspective is “useful for avoiding deficiency models of bilingual learners, developing detailed descriptions of the resources students use to communicate mathematically and helping teachers build on these resources during instruction.” This perspective also allows teachers to consider the students’ use of nonlanguage resources such as gestures and drawings.


This article synthesizes research by applied linguists and mathematics educators to highlight the linguistic challenges of mathematics and suggests pedagogical practices to help learners in mathematics classrooms. The linguistic challenges include the multi-semiotic formations of mathematics, its dense noun phrases, and the precise meanings of conjunctions and implicit logical relationships that link elements in mathematics discourse. Research on pedagogical practices supports developing mathematics knowledge through attention to the way language is used, suggesting strategies for moving students from informal, everyday ways of talking about mathematics into the registers that construe more technical and precise meanings.

Features of the classroom mathematics register:
- Multiple semiotic systems
- Mathematics symbolic notation
- Oral language
- Written language
- Graphs and visual displays
Grammatical patterns of the mathematics register:
- Technical vocabulary
- Dense noun phrases
- Being and having verbs
- Conjunctions with technical meanings
- Implicit logical relationships


The author provides strategies teachers can employ to support students in mathematical discussions. She answers research questions including: What can a teacher do to facilitate student participation in mathematical discussion? How can a teacher support students to speak mathematically? What are the variety of ways that students talk about mathematical objects? What are the different points of view students bring to mathematical situations? The author describes a mathematical discussion as “purposeful talk on a mathematical subject in which there are genuine pupil contributions and interactions” and suggests the following for supporting mathematical discussions:
- Model desired participation and talk; support these when displayed by students
- Encourage student conjectures and explanations
- Call for explanations and evidence for students’ statements
- Focus on the process not only the product
- Compare methods, solutions, explanations
- Engage students in arguments for or against a statement (move beyond “agree” or “disagree”)
- Encourage student-to-student talk
- Ask students to paraphrase each other’s statements
- Structure activities so that students have to understand each other’s methods

The article contains transcribed discussions with commentary highlighting aspects of mathematical discussions and how to support them.


This paper proposes a “sociocultural framework” to provide an integrated view of academic literacy in mathematics for English learners. The proposed definition of academic literacy in mathematics includes three integrated components: mathematical proficiency, mathematical practices, and mathematical discourse. The author advises that the three components not be separated in instruction and states that they are essential for mathematics instruction for English learners. The paper describes the benefits of this multi-faceted approach to developing mathematical literacy in that what students bring to the learning in each of the components is recognized and enhanced as competency and mathematical understanding is developed. The author provides classroom data from an eighth grade bilingual mathematics classroom that addresses how students displayed mathematical proficiency, communicated mathematically, and learned mathematical meanings within the socio-cultural context.
D. Effective, Culturally Relevant Mathematics Instruction for ELs


This monograph includes six articles related to mathematics instruction through a lens of social justice beginning with an interview with education scholar Danny Martin. Martin’s seminal work on race, racism, and the mathematics socialization of African Americans provides important understandings about social forces that impact the experiences and identities of people from a critical perspective. The second article provides an example of how young children can learn early numeracy concepts and critically examine historical civil rights struggles through play and storytelling. The third article describes a high school mathematics course interdisciplinary (language arts and mathematics) project with social justice goals utilizing the classic play, A Raisin in the Sun by Lorraine Hansberry. In the fourth article, the author describes her journey toward becoming a critical mathematics educator and describes how she implemented a mathematics activity with social justice implications with her bilingual preservice teachers and what she learned through the experience. The authors in the fifth article share their experience in a mathematics content course for preservice elementary teachers that use projects aimed at supporting students in the use of mathematics to understand their world within a social justice perspective. The author of the last article describes her professional acts of creative insubordination that center student advocacy in the teaching of mathematics.


The authors of this study acknowledge the utility of pedagogical tools that help teachers develop pedagogical knowledge. The authors identified the availability of tools to analyze and critique lessons around “pedagogical content knowledge” and “culturally responsive teaching,” but found that there was a need to consider “developing culturally responsive mathematics teaching” that combines the learning from each separate body of work. After examining the literature, a lesson analysis tool was created that incorporated eight dimensions of culturally responsive mathematics teaching:

- Intellectual support
- Depth of student knowledge and understanding
- Mathematical analysis
- Mathematical discourse and communication
- Student engagement
- Academic language support for ELs- use of L1
- Scaffolding strategies
- Funds of knowledge/culture/community support
- Use of critical knowledge/social justice.

The study used a qualitative research design with multiple data sources and focused on six beginning teachers. The lesson analysis tool was utilized to facilitate teacher planning, reflection, and professional learning. The use of the tool facilitated productive dialogue and self-reflection.

This resource guide was developed to provide scaffolding techniques and routines that are consistent with the Common Core State Standards and recent research. The guide begins with an overview of research-based scaffolding techniques for academic vocabulary; integrating oral and written language instruction into content area teaching; concrete and visual models; graphic organizers and foldables; multimedia to enhance comprehension; structured opportunities to speak with a partner or small group; regular, structured opportunities to write; sentence frames and starters; building background knowledge; background knowledge for teachers; clarifying content delivered in a second language; clarifying key concepts; teacher modeling and explanation; capitalizing on students’ home language and knowledge; side-by-side texts; and differentiating for students at diverse levels of English proficiency. The guide includes four sample lessons from the New York State Education Department that are annotated with scaffolding suggestions for English learners. The lessons provided are Kindergarten comparisons, fourth grade use of visual models to add and subtract fractions, eighth grade proofs of laws of exponents, and Algebra I power of exponential growth.


This paper discusses the relationship between mathematics learning and cognitive processes and resulting implications for ELs, the role of language structures in teaching and learning mathematics and implications for ELs, and proposed teaching and learning strategies for ELs. Related to cognitive processes, the author describes learning mathematics as learning new concepts and establishing new connections. ELs also face the challenge of translating new information from English to native language prior to forming connections to existing knowledge. Another potential difficulty for ELs may be a lack of informal knowledge to solve math problems. The author provides an example of a teacher who could not solve a math problem related to baseball because she lacked informal knowledge. Because students have a limited amount of memory available for storing new information, teachers should recognize that ELs may struggle with “extraneous linguistic information” that is not necessarily connected to the mathematical concept being presented. This may also happen during standardized testing. The author proposes a framework for teacher education courses that address four components of teaching mathematics to ELs: (a) academic content; (b) mathematical cognitive processes; (c) mathematical and contextual language; and (d) cultural/life experiences. Additionally, the author discusses the role of language structures in teaching and learning mathematics and implications for ELs and offers strategies for teaching ELs such as:

- Increased comprehensibility of mathematical presentation
- Focus on vocabulary and a scaffolded approach to presenting new facts
- Increased student-to-student interaction
- Connectivity between mathematics and the students’ experience
Encouraging code switching by students, use of multimodal teaching strategies, honoring the diversity of Latino students’ experiences, knowing the students and their experiences, avoiding deficit models, providing opportunities for mathematical discussions, and revoicing are also discussed.


This article describes instructional strategies utilized by teachers with at least 40% ELs to support ELs in understanding and participating in mathematical discourse. The authors remind readers that “If students are unable to access and participate in discourse, their opportunities to learn mathematics may be diminished.” The strategies are described and classroom examples from the researchers’ observations in fourth and fifth grade classes are included. The strategies are:

1. Ask open-ended questions of all students, including ELs
2. Scaffold ELs with close-ended questions, if needed
3. Scaffold responses by revoicing, repeating, rephrasing, and extending
4. Model vocabulary in context
5. Strive to engage ELs in discourse each day


This monograph focuses on issues related to diversity and equity in mathematics and contributes to the main goal of TODOS to advocate for an equitable and high quality mathematics education for all students, particularly Hispanic/Latino students. The monograph includes five chapters, or articles, that identify ways to recognize and build upon the resources of children, families, communities, and cultures for teaching mathematics to all students. The first article by Judit Moschkovich focuses on positioning and using students’ language as a resource with respect to specific mathematical practices. The second article by Anita Wagner and Kate Delaney illustrates how teachers can build on students’ strengths and interests to support their mathematics learning. The third article written by Erin Turner, Julia Aguirre, Tonya Bartell, Corey Drake, Mary Foote, and Amy Roth-McDuffie gives examples of how to connect children’s cultural funds of knowledge in mathematics lessons and provides specific lesson examples. The article by Rebecca Neal and Dan Battey attends to the kinds of relational interactions (moment-to-moment communication between teachers and students) can support and encourage students’ mathematical participation. In the last article, Higinio Domínguez focuses on moment-to-moment teacher-student interactions and recognizing resources generated by the student and teacher during mathematics instruction.
This article explains the importance of discussion routines to help English learners gain speaking skills, master academic content, and take charge of their learning. The authors find that students can concentrate on understanding math (or other content areas) because they are using a routine that is practiced often. Routines are critical for all students to participate and work collaboratively, rather than sit back and let other students talk or wait for the teacher to explain. The authors describe the use of routines in an “elbow partner exchange” and in “homework rounds.” Seven tips for using routines include:

1. Group students who have different strengths
2. Use simple, memorable routines
3. Use pressure to increase focus
4. Involve students in improving routines
5. Set rules that foster confidence
6. Stick with it
7. Assess students’ emerging skills

The authors of this article discuss the importance of promoting participation for English learners who might otherwise have few opportunities to practice the specialized language of mathematics and thus have inhibited opportunities to learn mathematics content. The authors remind teachers to: (1) support the development of mathematics and to plan to focus on the important mathematical ideas, tools, and representations that students should learn; (2) support the development of language by using strategies such as writing terminology on the board, asking students to produce writing, assisting students to revise their writing, and not avoiding mathematical vocabulary but supporting student use and understanding; (3) enhance mathematical tasks by analyzing the curriculum materials and consider adding and enhancing with visual supports, examining contexts of the learning that may be unfamiliar to ELs, and recognizing that explanations may be necessary to give ELs background knowledge to understand tasks and word problems; (4) establish, facilitate, and maintain productive classroom interactions that are critical for English learners in order to build their language skills, skills in expressing mathematical thinking, and skills for participating in collaborative and cooperative mathematics learning.
This framework was written in response to many urban districts reporting significant difficulty finding high quality, rigorous, grade-level instructional materials that are written for ELs at varying levels of language proficiency. The writers acknowledge that the influence of college- and career-ready standards and English language development (ELD) standards required by law have exacerbated the need for high quality instructional materials and instruction designed to support English learners to achieve deep understanding of mathematical concepts and simultaneously develop academic language proficiency.

The framework is divided into sections including: (a) making clear that the grade-level college- and career-readiness mathematics standards are for ALL students, including ELs; (b) articulating a theory of action that re-envision instruction for developing mathematics language and understanding; (c) identifying and providing instructional principles and practices designed to address the language demands in the new standards for mathematics; and (d) providing criteria for the selection of instructional materials for mathematics that attend to the academic language development and the language demands of the new standards for mathematical practices. The section on key instructional principles and practices defines proficiency as conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. The areas of instructional practice described include employing rigorous tasks and assignments, encouraging productive struggle, employing multiple modes and representations in mathematics, and using strategic scaffolding. For each instructional practice, classroom examples are provided. The framework would be useful for school-wide and grade level professional learning around instructional practices as well as for committees charged with selection of core and supplemental mathematics instructional materials.


This paper examines four research areas that challenge and add to the standard math taught in school with specific implications for Latino/a and black students’ achievement and opportunity in mathematics: ethnomathematics; how adults use math learning outside of school; afterschool math programs for youth; and social justice mathematics. Key findings include:

- The forms of mathematics U.S. schools value are not the only mathematics that people use. Different cultures practice different kinds of mathematics, learn them in different ways, and use them for different reasons.
- How students feel about themselves while doing mathematics is critical to how they engage in it. Out-of-school experiences help develop confidence, math strategies, and math identity.
- Small group learning, which is more common in afterschool programs, provides more opportunities for students to explain their thinking, get feedback quickly, and refine their thinking.
• Teaching mathematics through social justice issues can motivate adolescents -- especially those who have lost interest in traditional mathematics -- to learn math skills necessary to solve complex problems.


This author discusses the challenges and strengths of students participating in a multicultural mathematics classroom. The author reminds us that multicultural students come to the classroom with diverse cultural experiences including:

- Parental expectations
- Modes of communication
- Economic variations
- Use of different strategies for learning
- Cultural differences
- Rote memorization
- Home and school practices and discrepancies

The challenges educators face in providing culturally-based pedagogy are acknowledged and explained by recognizing the complexities of: acculturation and enculturation, communities of practices and the importance of supporting all students to be active participants, providing challenging tasks with carefully selected teaching practices, and breaking down stereotypes. The importance of positive interdependence amongst students, involving all students in the learning, and teacher and student use of multiple representations are presented along with the suggestion that teachers combine approaches when planning for multicultural mathematical instruction.


This article describes five questions to guide teachers as they teach mathematics and the language of mathematics simultaneously to students learning English. The questions include: “*Why are my EL students considered English learners?*” which prompts teachers to discover what languages and experiences their students bring with them to the learning. “*What mathematics do my English learners already know?*” addresses the need to know how much mathematical knowledge students have, in their native language as well as English. The article provides an example of different notation used in South American countries with division. Anticipating the differences in notation and making them explicit to students is important. “*How can I make my lessons more accessible to EL students (without simplifying the mathematical content)?*” requires teachers to analyze the language demands of the mathematical tasks and to consider students’ needs for scaffolding and support. The authors point out linguistic pitfalls and the critical importance of considering students’ background knowledge when analyzing tasks. “*How can I incorporate meaningful language practice into my lessons?*” requires teachers to provide more opportunities to speak and write about mathematics in English. The authors call for supporting students to develop academic language proficiency and “yielding the floor” to allow students to learn language within meaningful well-structured tasks. The final question, “*How can I leverage my
“classroom space to enhance student learning?” reminds teachers to make use of visuals, word walls, and charts for both the teachers and students to use during instruction.


This article discusses extensive research findings, principles and recommendations for supporting English learners in Common Core aligned mathematics instruction. Moschkovich begins by contrasting her view of “the language of mathematics” defined as “the communicative competence necessary and sufficient for competent participation in mathematical discourse practices” with less sophisticated views such as meanings of single words, proper use of grammar, or language used in a classroom. A more expanded, context-embedded view of the language required to engage in mathematical practices within a mathematics community allows teachers to plan for and provide high quality instruction that supports ELs that has two general characteristics: a view of language as a resource, rather than a deficiency; and an emphasis on academic achievement, not only on learning English. Research guidelines for instruction include providing students with:

- Teachers proven to be effective in supporting ELs academic success who have a high commitment to students’ academic success and to school-home communication, high expectations for all students, autonomy to change curriculum to meet the needs of diverse students, and a rejection of models of their students as intellectually disadvantaged
- Curricula with abundant and diverse opportunities for speaking, listening, reading and writing
- Instruction that encourages students to take risks, construct meaning, and seek reinterpretations of knowledge within compatible social contexts

The author highlights four Common Core emphases that must be taken seriously when considering the needs of English learners in mathematical instruction including balancing conceptual understanding and fluency, maintaining high cognitive demand, developing beliefs in students that mathematics is sensible, worthwhile and doable, and engaging students in mathematical practice. The author expands upon important recommendations for connecting mathematical content to language including: focusing on students’ mathematical reasoning, not accuracy in using language; shifting to a focus on mathematical discourse practices and moving away from a simplified view of language; recognizing and supporting students to engage with the complexity of language in math classrooms; treating everyday language and experiences as resources, not as obstacles; and uncovering the mathematics in what students say and do. Appendix A of the article provides a classroom vignette that provides descriptions of a lesson excerpt with commentary indicating how the instruction illustrates the recommendations played out in actual classroom instruction.


This professor in a school of education sought to answer the questions: *Given the proliferation of scholarship on culturally responsive teaching, why are teachers not engaged in culturally responsive mathematics practices?* and *What are specific culturally responsive teaching practices in mathematics?*
His participants were both pre-service and in-service teachers who enrolled in his graduate course on “Teaching and Learning in Urban Classrooms.” The author used structured in class activities, online discussions, and his own field notes to address the research questions. Four major themes were found to answer the question of what might prevent teachers from engaging in culturally responsive mathematical practices:

- View of mathematics as culturally neutral
- Convenience and dominance of textbook based mathematics instruction
- Curriculum standardization and high-stakes testing
- Lack of culturally responsive mathematics teaching models to emulate

Seven themes emerged to reflect culturally responsive mathematics teaching and each are explained with examples provided:

1. Deconstruct misguided beliefs about mathematics teaching and learning
2. Integrate culturally relevant content and social justice issues
3. Utilize culturally responsive instructional strategies
   a. Believe in students’ ability to learn and in their capability to do rigorous and high-level mathematics (high expectations)
   b. Provide instructional scaffolding that supports student success
   c. Know and care about students; positive teacher-student interactions and relationships in the learning community; promote cooperative, collaborative, and collective learning
   d. Contextualize teaching and learning by connecting what is taught to students’ lives and communities
   e. Engage in equitable and social justice practices; integrate students’ culture into the official curriculum
4. Foster communal learning
5. Be open to students’ divergent thinking and problem solving
6. Detrack the mathematics classroom
7. Encourage teacher critical consciousness, advocacy, and activism

The author suggests that teachers self-critique and question their own practice and provides reflective questions to help facilitate insight.


This document was created to provide guidance to mathematics teachers for recognizing and supporting students’ language development in the context of mathematical sense making. The theory of action is grounded in the “interdependency” of language learning and disciplinary learning, the central role of “student agency” in the learning process, the importance of “instructional scaffolding” that supports student participation, and the importance of “instructional responsiveness” in the teaching process. The framework includes four design principles for promoting mathematical language use and development of curriculum and instruction that make language a central part of planning and delivering instruction. The four principles below are explained in detail in the document:
• **Support sense-making**—scaffold tasks and amplify language so students can make their own meaning
• **Optimize output**—Strengthen the opportunities and supports for helping students to describe clearly their mathematical thinking to others, orally, visually, and in writing
• **Cultivate conversation**—strengthen the opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class)
• **Maximize linguistic and cognitive meta-awareness**—Strengthen the “meta-“ connections and distinctions between mathematical ideas, reasoning, and language

The mathematical language routines provided and described are based on the four design principles. The routines are effective for simultaneously learning mathematical practices, content, and language. The routines can be adapted and used across lessons and units. Following are the eight Mathematical Language Routines:

1. **Stronger and Clearer Each Time** provides a structured opportunity to revise and refine verbal and written output
2. **Collect and Display** captures students’ oral words and phrases into a stable, collective reference
3. **Critique, Correct, and Clarify** gives students a piece of mathematical writing that is not their own to analyze, reflect on, and develop
4. **Information Gap** creates the need for students to communicate
5. **Co-Craft Questions and Problems** allows students to get inside of a context before feeling the pressure to produce answers and creates a space for students to produce the language of mathematical questions themselves
6. **Three Reads** ensures that students know what they are being asked to do and creates opportunities for students to reflect on the mathematical questions being presented and equips students with tools used to negotiate meaning with math problems
7. **Compare and Connect** fosters students’ meta-awareness as they identify, compare, and contrast different mathematical approaches, representations, concepts, examples, and language
8. **Discussion Supports** facilitate rich and inclusive discussions about mathematical ideas, representations, contexts, and strategies

Taken together, the framework principles and mathematical language routines provide the research support and actual classroom practices that afford all students the opportunity to develop mathematical understanding and mathematical language simultaneously.

**E. The Role of Instruction, Professional Learning, Teacher Collaboration and Coaching Support**


This article discusses the dearth of research on what constitutes a highly qualified teacher of ELs but points to some important characteristics:
• **Bilingualism**
The article points to the urgent need for professional development for teachers who teach English learners. One study cited surveyed 5,300 educators in California and found that among the teachers with 50 percent or more ELs, about half had either no professional development or only one session in over a five-year period. A more recent study of more than 550 teachers in Los Angeles revealed that 17 hours per year on average were focused on professional development for teachers teaching ELs, yet the teachers did not view the amount as sufficient. There is a strong need for teacher preparation programs to train teachers of ELs and for districts to provide sufficient professional development and support for all teachers who teach ELs. While the recruitment of bilingual teachers was mentioned as a solution, the authors do not believe that there will be enough bilingual teachers in the foreseeable future. Teachers surveyed are looking for strong leaders/principals to provide time and support as well as schoolwide efforts to create school systems to support ELs.


This paper describes specific elements of effective practice that need to be incorporated into all comprehensive teacher preparation and school reform efforts. The paper identifies four essential elements of EL program implementation:

- **Rigorous and relevant instructional practices** that: maximize the development of English while simultaneously maximizing the development of content knowledge; value the students’ home language and culture; bridge connections from students’ prior knowledge and experiences; provide cycles of input, clarifications and use of questioning strategies as well as support for native language development; and structure classrooms to support multiple opportunities for interaction and student academic language interactions
- **Multiple measures for English learner assessment** by principals and teachers who frequently assess student progress in all areas of instruction, use the data to address the academic needs of students, and connect assessment results to inform teaching practices
- **Assessment of practices of teachers of English learners** using fair, reliable and valid multiple measures to determine teacher effectiveness. Observational protocols should be standards-aligned and measure teacher practices with ELs across content areas.
- **Collaboration and professional development** that provides teachers with time to reflect and collaborate on instructional practices and utilize assessment information to inform instruction. Strong systems of classroom based professional development should be provided to teachers and leaders. The expert teacher is defined as a knowledgeable, professional who is...
accomplished in curriculum, linguistics, cross-cultural understanding, assessment, and student advocacy.


This white paper focuses on understanding the importance of mathematical discourse and discusses how to establish a discourse-rich mathematics learning community, provides ways to engage every student in mathematical discourse, and reminds us that all students are mathematics language learners that must engage in discourse to practice vocabulary, symbols, syntax and semantics. The paper describes strategies for establishing classroom environments that support mathematical discourse:

1. Help students work together and rely on one another
2. Allow students to work independently before sharing in small or large groups
3. Use questions strategically to engage students in mathematical discourse
4. Acknowledge the importance of mistakes in learning and understanding
5. Use collaborative learning strategies to support students in preparation for whole-class discussions
6. Use a variety of pedagogical strategies to engage all students in whole-class, teacher-led mathematical discussion

The paper includes questions teachers can utilize to promote mathematical thinking and discourse that are organized by purpose: helping students work together to make sense of mathematics, helping students persevere, helping students to learn to reason mathematically, helping students to evaluate their own processes, helping students with comprehension, helping students learn to conjecture, invent, and solve problems, helping students to connect mathematics, its ideas and applications, helping students rely on themselves to determine whether something is mathematically correct, and helping students focus on mathematics from activities.


This paper describes the teacher development necessary to build the knowledge and skills of all teachers to provide instruction in the Common Core State Standards that require new and deeper academic and language demands. In mathematics, the standards envision:

- Problem situations that are language-rich and require multiple steps
- Concepts represented in multiple ways
- Procedures that constitute a special narrative

The implications for teacher development call for Institutions of Higher Education (IHE), school districts, and other partners to build the capacity of teachers to serve all students, with special attention to the growing population of ELs. The authors propose that educators will need:
To understand the shifts required in curriculum, instruction, and assessment and hands-on professional learning opportunities to acquire teaching strategies that respond to the shifts

• Deep content knowledge that is pedagogical in nature
• To understand deeply the core areas of the disciplines and the learning progressions within
• To know a great deal about formative assessment to understand current student understanding in relation to the standards
• A variety of curriculum and instructional supports to respond to students’ needs

To meet the specific needs of English learners, teachers need to know how to address:

• **Language progressions** - how students learn language, both in terms of general language acquisition and discipline-specific academic language acquisition
• **Language demands** - what kinds of linguistic expectations are embedded within specific texts and tasks
• **Language scaffolds** - how specific strategies can be used to help students gain access to the concepts as well as to the language they need to learn
• **Language supports** - how classrooms and schools can be organized to support students in continually building a deep understanding of language and content

The paper describes the pre-service preparation and professional learning opportunities that will support teachers to develop understanding of content pedagogy that incorporates an understanding of the language of the disciplines. The authors suggest models of teacher support, cross-role and job-alike teams, cross-disciplinary teams, school cycles of inquiry, collaborative networks, and sustained subject matter networks to support problems of practice and teacher learning.


This framework provides a research-based answer to the question, “What are the attributes of equitable and robust learning environments- environments in which all students are supported in becoming knowledgeable, flexible, and resourceful disciplinary thinkers?”

The quality of the learning environment depends on the extent to which it provides opportunities for students in five dimensions:

1. The richness of disciplinary concepts and practices (“the content”) available for learning
2. Student sense-making and “productive struggle”
3. Meaningful and equitable access to concepts and practices for **all** students
4. Means for constructing positive disciplinary identities through presenting, discussing, and refining ideas
5. The responsiveness of the environment to student thinking

The framework, along with a collection of tools for teachers, coaches, and administrators, support the development of deeper understandings of teaching and how to enrich it. The comprehensive set of
tools available includes this introduction to the framework, conversation guides for reflective planning and teaching, observation guides, and rubrics. Most of the tools are available in mathematics-specific and domain-general versions and are available at: [http://map.mathshell.org/trumath.php](http://map.mathshell.org/trumath.php)


This R & D Connection provides an overview of statistics on achievement, identification of gaps in test scores between student groups, and a reminder that “the entire educational community is now faced with the challenge of how to best address ELs’ educational needs.” Recommendations for training for teaching content standards to English learners and improving teaching of content to English learners are described including the role of language in teaching content and what content teachers need to know. Examples that illustrate the role linguistic features can have on content learning are provided in the form of a math word problem and text excerpt. The authors provide implications for teacher training and suggestions for future research.
1. Curricular resources for Common Core mathematical approaches

Below is a list of websites that represent several of the main professional associations and instructional approaches that are currently available to districts and teachers. These resources were selected based on the frequency with which they were referenced in the academic and professional literature cited in the Annotated Bibliography and by the expert interviews conducted for the *Unlocking Learning II: Math as a Lever for English Learner Equity* report. The list is intended as a resource to inform educators of what is available, not as an endorsement of any particular curriculum, organization, or product. Along with each site listed is a short summary of the instructional approach and resources available on the website, as well as highlights of any resources that appear to be particularly useful for teachers and instructional leaders.

**Algebra by Example**
[http://math.serpmedia.org/algebra_by_example/](http://math.serpmedia.org/algebra_by_example/)

Developed to improve achievement in Algebra 1, this set of 42 assignments considers research that suggests that the simple instructional technique of having students study examples of problems and solutions and explain targeted correct or incorrect steps in the example improves students’ conceptual knowledge without sacrificing procedural skill. These assignments can supplement any math curriculum.

**Tools for Sense-making in Mathematics**
[http://math.serpmedia.org/sense-making/index.html](http://math.serpmedia.org/sense-making/index.html)

This website is the product of a SERP collaboration with middle school mathematics teachers in the San Francisco Unified School District. Over the course of several years, Phil Daro (SERP Bay Area Math Director and co-author of the Common Core State Standards in Mathematics), Professor Alan Schoenfeld (U.C. Berkeley), and a team of teachers and graduate students met monthly to explore together why students were struggling with math in the middle grades, and what they could do to support student learning. The initial data indicated students did not struggle with particular math topics, but rather with particular problem types: word problems. The tools developed to support students with mathematical word problems include: Stem/Question/Solution Triangles, Using Multiple Representations, Mathematical (and not so mathematical) Diagrams, and Curriculum for Diagnostic Teaching.
2. Free online tools for math teachers

**WordSift**
https://wordsift.org/

WordSift is a free online tool designed in collaboration with middle school teachers to address vocabulary and academic language in content area instruction. It was developed by Kenji Hakuta at Stanford University and is intended to support English learners.

**The SERP 5x8 Card to Facilitate Classroom Observations of Mathematics Lessons**
http://math.serpmedia.org/5x8card/
http://math.serpmedia.org/5x8card/deck/

The SERP (Strategic Education Research Partnership) 5x8 card was developed to help focus principals’ observations in math classrooms on the implementation of the CCSS mathematical practices. Principals, teachers, teacher leaders, and coaches have embraced the tool to support teachers making the shifts to support students in the mathematical practices. The “Deck behind the 5x8 Card” offers further research, explanation, and examples to deepen and refine actual classroom implementation.

**Illustrative Mathematics**
https://www.illustrativemathematics.org

This site provides high quality, rigorously reviewed tasks from teacher leaders across the nation to illustrate the Common Core standards for mathematics. The site contains curriculum, professional learning, content standards, practice standards, blueprints, progressions, and the recently released Illustrative Mathematics Middle School Curriculum designed to include adaptations and scaffolding support for English learners.

3. Resources for professional learning about ELs and mathematics instruction

**Shifting Course Pathways to Align with the CCSS-M**
http://serpinstitute.org/assets/daro_serpro_ccss_and_acceleration.pdf

The Common Core State Standards in Mathematics specify content to be taught to eighth grade students that is not aligned with a typical Algebra 1 course. In districts that have moved Algebra 1 to eighth grade, the standards come into conflict with what has become the symbol of a rigorous mathematics course pathway. SERP collaborated with the math leadership teams in San Francisco Unified School District and Oakland Unified School District to generate course pathways that preserve the CCSS-M focus on eighth grade math content, but allow students to compress courses in order to complete as many math courses as were possible when eighth grade Algebra 1 was the norm. The course pathway proposals and their presentation to the school boards in the two districts model effective negotiation of the concerns of parents for college preparatory instruction and the knowledge of mathematics specialists regarding critical middle school mathematics that should not be skipped in the interest of acceleration.
Mathematics Improvement Network
http://mathnic.org

The Mathematics Network of Improvement Communities (Math NIC) has developed nine prototype tools to assist in the improvement of mathematics programs and instruction. The topics include Teaching for Robust Understanding, Lesson Study for Professional Development, Developing Mathematical Proficiency, Formative Assessment, and Mathematical Practices, among others. The (free) materials are designed to support workshop-style professional learning sessions and each module (designed to be 90-120 minutes) includes a leader guide, powerpoint slides, handouts, and videos.

Youcubed
https://www.youcubed.org

This Stanford center provides mathematics education resources to teachers, students, and parents that support “open, creative mathematics.” The site includes tasks, videos, courses, evidence, and books on topics such as visual mathematics, growth mindset, depth not speed, brain science, and group work.

Mathematics Assessment Project
http://map.mathshell.org/

The Mathematics Assessment Project set out to design and develop well-engineered tools for formative and summative assessment that expose students’ mathematical knowledge and reasoning, helping teachers guide them towards improvement and to monitor progress. The tools are relevant to any curriculum that seeks to deepen students’ understanding of mathematical concepts and develop their ability to apply that knowledge to non-routine problems. The materials provided include summative tests or tasks and classroom challenges.

Understanding Proficiency
https://understandingproficiency.wested.org

This website was developed by a partnership between WestEd and SCALE. The website provides student work samples scored and annotated by teachers on performance tasks in mathematics and ELA/literacy in grades 3-8 and high school. The site additionally provides professional development tools with suggested activities for using the resources, including videos of educators scoring and discussing student work.

California Mathematics Project
http://www.cmpso.org

The California Mathematics Project provides formal partnerships with schools and/or districts to deliver services to teachers of mathematics in high-poverty and diverse schools. The project provides English Language Development (ELD) assistance and materials that support
mathematics and literacy and identifies, develops, and sustains mathematics teacher leaders in education communities, and expands statewide opportunities for professional development, amongst other goals. The project supports 19 regional sites located on university campuses.

**Teaching High School Math Using SDAIE**
[http://web.stanford.edu/dept/gse/cgi-bin/clad/elr012/](http://web.stanford.edu/dept/gse/cgi-bin/clad/elr012/)

This collection of resources includes videos, assignments, and links to articles and information that discuss the multiple ways in which language develops and is present in math classrooms. Educators are encouraged to incorporate Specially Designed Academic Instruction in English (SDAIE) strategies into instruction in order to address both content and language objectives effectively within the same math lesson.

**Colorín Colorado**

Colorín Colorado is a national multimedia project that offers research-based information, activities, and advice for educators and families of ELs. The website’s page on mathematics instruction for ELs offers strategies, resources, and guidance for helping ELs succeed in mathematics.

**TODOS: Mathematics for ALL-Excellence and Equity in Mathematics**

TODOS is an organization affiliated with the National Council of Teachers of Mathematics that operates on the belief that “we must work to create a more just, humanizing, and equitable mathematics education experience for all.” The website has publications, parent and family resources, and information about TODOS events. With the purchase of a membership ($25 per year) members have access to additional resources.

**Understanding Language Initiative at Stanford University (Hakuta & Santos)**

*Understanding Language: Language, Literacy, and Learning in the Content Areas* is a collective of experts in the field of language and content area teaching and learning, currently led by Hakuta and Santos. Understanding Language aims to develop research and resources to increase awareness of how essential it is for all students, but especially English learners, to learn the language of each academic discipline so they can rigorously and authentically engage with its content, as is intended by the CCSS. A collection of CCSS-aligned mathematics tasks with annotations and other resources are included in the download “Supporting ELLs in Mathematics.”

**California Common Core State Standards Mathematics**
California Mathematics Framework Chapters
http://www.cde.ca.gov/ci/ma/cf/mathfwchapters.asp

California Mathematics Framework Chapter: Universal Access

This chapter provides information on differentiation, Universal Design for Learning, language demands of the CA CCSSM, assessment, accommodations, assistive technology, and beginning on page 683, planning instruction for California’s English learners. Mathematics discourse, meeting the needs of long-term English learners, and planning instruction for English learners are discussed.

California Common Core State Standards Mathematics Resources
https://www.mydigitalchalkboard.org/portal/default/Resources/CollectionViewer/CollectionViewer?action=2&id=501332

This page contains 28 links to resources for teaching the Common Core Mathematics Standards including Engage New York, Achieve the Core, lessons, videos, professional learning modules, and a wide variety of resources for supporting the implementation of effective instruction.

English Language Development Standards:
https://www.wida.us/standards/eld.aspx (national)
http://www.cde.ca.gov/sp/el/er/documents/eldstndspublication14.pdf (California)

Integrating the CA ELD Standards into K–12 Mathematics and Science Teaching and Learning
http://www.cde.ca.gov/sp/el/er/eldstandards.asp

This resource specifies the correspondences between the CA ELD Standards and the CA Common Core Math Standards and provides illustrative examples of the implementation of the CA ELD Standards in tandem with the CA CCSSM and the CA NGSS. It is designed as a supplementary resource to the California curriculum frameworks for English language arts/English language development (ELA/ELD), mathematics, and science, as well as to the CA ELD Standards, CA CCSSM, and CA NGSS documents themselves.

Recommended books & manuals
(Listed from newest to oldest date of publication; book summaries provided by the publishers.)

This National Academies of Sciences report provides insights for new directions for researchers, policymakers and practitioners to better support dual language learners (DLLs) and English Learners (ELs). The report provides knowledge about the cultural and linguistic assets of ELs, as well as the challenges faced in supporting their educational success. It also has specific recommendations to improve the preparation, competency, and practice of the early care and education workforce serving DLLs and ELs.


*Mathematical Mindsets* provides practical strategies and activities to help teachers and parents show all children, even those who are convinced that they are bad at math, that they can enjoy and succeed in math. Jo Boaler—Stanford researcher, professor of math education, and expert on math learning—has studied why students don’t like math and often fail in math classes. She’s followed thousands of students through middle and high schools to study how they learn and to find the most effective ways to unleash the math potential in all students.

There is a clear gap between what research has shown to work in teaching math and what happens in schools and at home. This book bridges that gap by turning research findings into practical activities and advice. Boaler translates Carol Dweck’s concept of ‘mindset’ into math teaching and parenting strategies, showing how students can go from self-doubt to strong self-confidence, which is so important to math learning. Boaler reveals the steps that must be taken by schools and parents to improve math education for all.


This book addresses a gap acknowledged in the Common Core State Standards for Mathematics (CCSSM): “It is ... beyond the scope of the Standards to define the full range of supports appropriate for English language learners.” Authors of the ten chapters that compose this collection focus on pedagogical practices that can support ELLs and their teachers in meeting the content and language demands of CCSSM. Each chapter features detailed classroom-based vignettes that highlight specific pedagogical practices that teachers can use to support ELLs in building their skill with the practices identified in the Standards for Mathematical Practice, and each concludes with questions for reflection and suggestions for action plans. CCSSM has been adopted by most states in the country, and mathematics educators must attend to the needs of language learners as they respond to this widespread adoption, implement the new standards in their classrooms, and ensure their students’ success.
English language learners share a basic need—to engage, and be engaged, in meaningful mathematics. Through guiding principles and instructional tools, together with classroom vignettes and video clips, this book shows how to go beyond good teaching to support ELLs in learning challenging mathematics while developing language skill.


Issues of language in mathematics learning and teaching are important for both practical and theoretical reasons. Addressing issues of language is crucial for improving mathematics learning and teaching for students who are bilingual, multilingual, or learning English. These issues are also relevant to theory: studies that make language visible provide a complex perspective of the role of language in reasoning and learning mathematics. What is the relevant knowledge base to consider when designing research studies that address issues of language in the learning and teaching of mathematics? What scholarly literature is relevant and can contribute to research? In order to address issues of language in mathematics education, researchers need to use theoretical perspectives that integrate current views of mathematics learning and teaching with current views on language, discourse, bilingualism, and second language acquisition. This volume contributes to the development of such integrated approaches to research on language issues in mathematics education by describing theoretical perspectives for framing the study of language issues and methodological issues to consider when designing research studies. The volume provides interdisciplinary reviews of the research literature from four very different perspectives: mathematics education (Moschkovich), Cultural-Historical-Activity Theory (Gutiérrez, Sengupta-Irving, & Dieckmann), systemic functional linguistics (Schleppegrell), and assessment (Solano-Flores).