



**A Pathway to
Equitable
Math Instruction**
**SEAD Theme
Guidebook:
Discourse**

STRIDE

3

Discourse SEAD Theme Guidebook

Discourse for Mathematics Grade 6–8

NCTM Definition of Discourse: *“ways of representing, thinking, talking, agreeing, and disagreeing; the ways ideas are exchanged and what the ideas entail; and as being shaped by the tasks in which students engage, as well as by the nature of the learning.”*

The Collaborative for Academic, Social and Emotional Learning (CASEL) defines the characteristics of Discourse:

- **Self-Awareness:** The ability to accurately recognize one’s own emotions, thoughts, and values and how they influence behavior. The ability to accurately assess one’s strengths and limitations, with a well-grounded sense of confidence, optimism, and a “growth mindset.”
- **Social Awareness:** The ability to **take the perspective of** and empathize with others, including those from diverse backgrounds and cultures. The ability to understand social and ethical norms for behavior and to recognize family, school, and community resources and supports.
- **Responsible Decision-Making:** The ability to make constructive choices about personal behavior and social interactions based on ethical standards, safety concerns, and social norms. The realistic evaluation of consequences of various actions, and a consideration of the well-being of oneself and others.
- **Self-Management:** The ability to successfully regulate one’s emotions, thoughts, and behaviors in different situations—effectively managing stress, controlling impulses, and motivating oneself. The ability to set and work toward personal and academic goals.
- **Relationship Skills:** The ability to establish and maintain healthy and rewarding relationships with diverse individuals and groups. The **ability to communicate clearly, listen well, cooperate with others**, resist inappropriate social pressure, **negotiate conflict constructively, and seek and offer help when needed.**

In addition, CASEL describes the characteristic of discourse *in mathematics* as also including:

- Encouraging student academic talk in mathematics instruction.
- Increasing student talk time so it is balanced with, or exceeds, teacher talk within lessons.
- Allowing for opportunities for students to understand the viewpoints of others, including both/multiple sides of an issue.

Standards for Mathematical Practice¹

SMP 1: Make sense of problems and persevere in solving them.

(STRONG CONNECTION WITH DISCOURSE)

Mathematically proficient students start by **explaining to themselves** the meaning of a problem and looking for entry points to its solution. They **analyze** givens, constraints, relationships, and goals. They **make conjectures** about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt....Mathematically proficient students can **explain correspondences** between equations, verbal descriptions, tables, and graphs, or draw diagrams of important features and relationships, graph data, and search for regularity or trends....Mathematically proficient students check their answers to problems using a different method, and they continually **ask themselves**, “Does this make sense?” They can **understand the approaches of others** to solving complex problems and **identify correspondences** between different approaches. (Excerpted “Overview of Standards”².)

¹ “Overview of the Standards Chapters of the Mathematics Framework for California Schools,” California Department of Education, 2015, <https://www.cde.ca.gov/ci/ma/cf/documents/mathfwoverview.pdf>.

² “Overview of the Standards Chapters”

Sample Actions / Instructional Strategies Related to Discourse and SMP 1

Example referenced from [Student Achievement Partners document](#):

- ▼ Communicate that students' thinking is valued—build trust and rapport by asking questions that elicit students' thinking, such as when students are analyzing proportional relationships.

Examples referenced from [ELSF Mathematics Guidance](#):

- ▼ Use Three Reads strategy for students to communicate about story problems.
- ▼ Use observation cards strategy to help students discuss their ideas and arguments.
- ▼ Use manual cues (hand motions) strategy to help students discuss their ideas and arguments.

Examples referenced from ["Supporting ELLs in Mathematics"](#):

- ▼ Use Math Talk to Improve the quality of discourse in mathematics conversations (*Classroom Discussions: Using Math Talk to Help Students Learn*).

Example referenced from [Teachers Development Group](#):

- ▼ Invite a group of students to discuss the mathematical conditions in which an idea will or won't always work.

Student Actions ³	Teacher Actions ⁴
<ul style="list-style-type: none"> ● Listen to the reasoning of others, compare arguments, and decide whether the arguments make sense. ● Ask clarifying and probing questions. ● Engage in discussions about the math concepts and processes. 	<ul style="list-style-type: none"> ▼ Create a culture that welcomes “rough draft” thinking. ▼ Provide opportunities for academic conversations, with sentence starters or frames. Use student talk to clarify ideas and make connections to other students’ responses. ● Avoid giving too much assistance (e.g., providing answers or procedures). ● Highlight the ways that one student’s thinking influences others.

³ [Examples referenced and extended from Kansas Math Standards Flipbook](#)

⁴ [Examples referenced and extended from Kansas Math Standards Flipbook](#)

Summary of Practice / Best Practices / Reflection for Discourse and SMP 1

- ▼ Interpret and make meaning of the problem, looking for starting points. Analyze what information is already given to have students explain to themselves/each other the meaning of the problem in their own words. Let them own the problem.
- Plan a solution pathway with the students' help, instead of jumping to a solution.
- ▼ Identify language demands and provide the language supports that ELs will need to understand the content, such as audio recordings, highlighted key concepts in the text, and modifying the text-based questions if necessary.
- ▼ Engage students in open-ended math discussions. Use mathematics and language development routines to cultivate discussions in pairs and small groups to maximize each EL student's opportunity to use language for academic purposes.
- Relate current situations to concepts or skills previously learned and connect mathematical ideas to one another. This provides students an easier opening to be able to talk about the ideas.
- Continually ask themselves, "Does this make sense to me?"
- ▼ Don't forget all the meaning-making resources and realia that you use in the regular classroom; similar tools are available online and in students' immediate environment. Tools such as virtual manipulatives, Desmos, and GeoGebra provide ELs with additional, dynamic sense-making opportunities.

- ▼ This icon is used to identify strategies and actions particularly beneficial to support ELs in mathematics because they integrate language and content.

Standards for Mathematical Practice⁵

SMP 2: Reason abstractly and quantitatively.

(SOME CONNECTION WITH DISCOURSE)

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to **decontextualize**—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to **contextualize**, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a **coherent representation** of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. (Excerpted from NCTM definition.)

⁵ “Overview of the Standards Chapters”

Sample Actions / Instructional Strategies Related to Discourse and SMP 2

Example referenced from [Student Achievement Partners document](#):

- ▼ Build a safe community where mathematical discourse supports active listening, promotes diverse perspectives and insights, and allows students to consider others' reasoning to advance their own mathematical understanding.
- ▼ Utilize a "which one doesn't belong?" activity for groups of students to discuss and analyze correspondences between graphs, tables, and equations that represent a relationship between dependent and independent variables.

Examples referenced from ["Supporting ELLs in Mathematics"](#):

- ▼ Provide visual supports to explain complex ideas. These supports should be presented to EL students and discussed by them before asking them to use one independently.
- ▼ Invite EL students to demonstrate their reasoning at home using *realia* (e.g., using simple measuring tools found at home).
- ▼ Use a Barrier Game for students to be able to represent their side of the shared problem or story.

Student Actions	Teacher Actions
<ul style="list-style-type: none"> ● Use varied representations and approaches when solving problems. ● Represent situations symbolically and manipulate those symbols easily. ● Use audio recordings to represent EL student explanations instead of a written response. ● Give meaning to quantities (not just computing them) and making sense of the relationships within problems. 	<ul style="list-style-type: none"> ▼ Ask students to explain the meaning of the symbols in the problem and in their solution. ▼ Expect students to give meaning to all quantities in the task. Identify the uses of English that may be confusing and highlight and use cognates when applicable. ● Question students so that the relationships between the quantities and/or the symbols in the problem and the solution are fully understood. ● Provide a teacher-completed graphic organizer to highlight key concepts about relationships. Walk students through your process to complete the organizer.

Summary of Practice / Best Practices / Reflection for Discourse and SMP 2
<ul style="list-style-type: none"> ● Make sense of quantities and their relationships. ● Decontextualize (represent a situation symbolically and manipulate the symbols) and contextualize (make meaning of the symbols in a problem) quantitative relationships. ● Understand the meaning of quantities and be flexible in the use of operations and their properties. ▼ Create a logical representation of the problem, and if possible use visual models to assist in understanding. If possible, keep these up on the wall for student use. ▼ Learning activities for ELs in a virtual environment might include the use of digital manipulatives and opportunities for students to draw, save, view, and comment on mathematical representations (e.g., diagrams, numbers lines, graphs, formulas, and equations). ▼ Even if an EL student has not been taught with his/her primary language, there is vocabulary from the home that is an asset to learning mathematical terms. Consider using resources from spanishcognates.org or finding a cognate dictionary for the native language of your students.

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Standards for Mathematical Practice⁶

SMP 3: Construct viable arguments and critique the reasoning of others.

(STRONG CONNECTION WITH DISCOURSE)

Mathematically proficient students understand and use **stated assumptions, definitions**, and previously established results in **constructing arguments**. They **make conjectures** and **build a logical progression of statements** to explore the truth of their conjectures. They are able to **analyze situations** by breaking them into cases, and can recognize and **use counterexamples**. They **justify their conclusions, communicate them to others, and respond to the arguments of others**....Mathematically proficient students are also able to **compare the effectiveness of two plausible arguments**, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—**explain what it is**....Students at all grades can **listen or read the arguments** of others, decide whether they make sense, and **ask useful questions** to clarify or improve the arguments. (Excerpted “Overview of Standards”⁷.)

⁶ “Overview of the Standards Chapters”

⁷ “Overview of the Standards Chapters”

Sample Actions / Instructional Strategies Related to Discourse and SMP 3

Example referenced from [Student Achievement Partners document](#):

- ▼ Use discussion protocols to provide a safe environment for students to share their developing thinking and to allow for interactions where peers value multiple contributions, such as Socratic Seminar and Fishbowl.

Examples referenced from [ELSF Mathematics Guidance](#):

- ▼ Choose and Defend is a helpful protocol to build up the routinized conversation segments for ELs.
- ▼ Group Activity: Problem Solving Quadrant is a barrier game for building the vocabulary and content necessary to construct viable arguments.

Examples referenced from other sources:

- ▼ Consider using the [World of Tales](#) to incorporate culturally relevant materials, providing important linkages between the home language and school language.
- ▼ Design math discussions for student-led small groups using platforms such as Zoom, Google Meet, or a Jamboard. Prompts for extensive discussion for ELs might include: *Do you agree with the strategy or explanation? How would you explain your strategy to others?*

Student Actions	Teacher Actions
<ul style="list-style-type: none"> ● Make conjectures and explore the truth of those conjectures by talking them over with another student. ● Recognize and use counter examples. Construct a t-chart so that students can visually see the differences. ● Justify and defend all conclusions by using data within those conclusions. ● Recognize and explain flaws in arguments, which may need to be demonstrated using objects, pictures, diagrams, or actions. 	<ul style="list-style-type: none"> ● Encourage students to use proven mathematical understandings (definitions, properties, conventions, theorems, etc.) to support their reasoning. ● Question students so they can tell the difference between assumptions and logical conjectures. ● Ask questions that require students to justify their solution and their solution pathway. ▼ For EL students, create short slide presentations to explain and extend the content in a lesson about constructing arguments. Provide sentence stems for students to use in their responses, such as <ul style="list-style-type: none"> ▪ Analysis: “How would you explain...? What is the importance of...?” ▪ Clarification: “Explain how... What is meant by...?” ▪ Cause and Effect: “What connection is there between ...?” ▪ Comparison: “What is the difference between...? How are they alike?” ● Ask students to compare and contrast various solution methods and to respectfully evaluate peer arguments.

Summary of Practice / Best Practices / Reflection for Discourse and SMP 3

- Analyze problems and use stated mathematical assumptions, definitions, and established results in constructing arguments.
- Justify conclusions with mathematical ideas, using support from anchor charts and other visual aids.
- ▼ Listen to the arguments of others and ask useful questions to determine if an argument makes sense. Ask clarifying questions or suggest ideas to improve/revise the argument.
- Compare two arguments and determine correct or flawed logic.
- ▼ Use multilingual resources that can “refresh” EL students with concepts they have already learned but may have forgotten. These include posters about content in Spanish or other primary languages, online flashcards, YouTube videos and other sources.
- Online discussion boards can keep the conversation alive over time. Pose questions such as,
 - What made you think that?
 - Can you give me another example so I understand, or an example from the problem itself?

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Standards for Mathematical Practice⁸

SMP 6: Attend to precision.

(STRONG CONNECTION WITH DISCOURSE)

Mathematically proficient students try to **communicate precisely** to others. They try to **use clear definitions** in discussion with others and in their own reasoning...By the time they reach high school they have learned to **examine claims** and make **explicit use of definitions**. (Excerpted “Overview of Standards”⁹.)

⁸ “Overview of the Standards Chapters”

⁹ “Overview of the Standards Chapters”

Sample Actions / Instructional Strategies Related to Discourse and SMP 6

Examples referenced from [ELSF Mathematics Guidance](#):

- ▼ Consider the range of student English language proficiency levels in your class and the language needed to access the content. Use online tools that include hyperlinked academic language connected to glossaries, examples, videos, or interactive organizers to provide scaffolding in both synchronous and asynchronous environments.
- ▼ For ELs, take time to develop a Frayer Model definition for each of the required terms with the students so the definitions and uses are clear.
- ▼ Develop a set of bounce cards for students to use to practice terminology.
- ▼ Choose a Reading and Understanding the Problem activity to give students time to use the vocabulary of the problem.

Student Actions	Teacher Actions
<ul style="list-style-type: none"> ● Use mathematical terms, both orally and in written form, appropriately, using visual supports as needed. ● Use and understand the meanings of math symbols that are used in tasks. ● Calculate accurately and efficiently. ● Understand the importance of the unit in quantities. 	<ul style="list-style-type: none"> ▼ Consistently use and model correct content terminology. Provide sentence frames or starters for ELs to model the appropriate phrasing. ▼ Expect students to use precise mathematical vocabulary during mathematical conversations and support them by having anchor charts, visuals, word walls, etc., with the desired terms. (ELSF Mathematics Guidance) ● Question students to identify symbols, quantities, and units in a clear manner, allowing them to practice with partners during informal times.

Summary of Practice / Best Practices / Reflection for Discourse and SMP 6

- ▼ Present academic terms through pre-teaching, using word cards, posters, student-made dictionaries, and by using activities such as Dictogloss. Vocabulary/precise terminology provides a challenge to ELs in that they have double the work—they must master both the content and the language of the content standards. ([ELSF Mathematics Guidance](#))
- Communicate precisely with others and try to use clear mathematical language when discussing their reasoning. Some student discussion can be written on charts, with the class suggesting more clear and precise language for improvement.
- Understand meanings of symbols used in mathematics and label quantities appropriately.
- Express numerical answers with a degree of precision appropriate for the problem context.
- Calculate efficiently and accurately.

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Example of connections to priority mathematics content standards:

Students can develop **Discourse** as they engage in a lesson related to “*Draw informal comparative inferences about two populations*” (7.SP.B). This can be accomplished by supporting students in SMP 2 utilizing the following actions and strategies:

- ▼ Provide visual supports to explain complex ideas. These supports should be presented to EL students and discussed by them before asking them to use one independently.
- ▼ Ask students to explain the meaning of the symbols in the problem and in their solution.
- Question students so that the relationships between the quantities and/or the symbols in the problem and the solution are fully understood.

Glossary for Discourse (from SMPs 1, 2, 3, 6)

Analyze situations and problems: *to study (something) closely and carefully; to learn the nature and relationship of the parts of (something) by a close and careful examination.*

Ask useful questions to **clarify** or improve arguments/statements: *to make (something) clear or clearer, such as to make (something) easier to understand.*

Build **logical progressions** of statements: *to build ideas that agree with the rules of logic, that are sensible or reasonable, and that lay out a continuous and connected series of actions, events, etc.*

Compare arguments, recognize and **explain flaws**: *to look at (a statement or series of statements for or against something) closely in order to see what is similar or different about them or to decide which one is better, and be able to explain small faults or weaknesses.*

Communicate precisely: *to be very careful and exact about the details of something that a person is talking about.*

Construct arguments and represent them **coherently**: *to make or create (something, such as an argument) by organizing ideas, words, data, etc., and to describe the argument so that it makes sense to those who are listening.*

Contextualize and decontextualize: *to think about or provide information about the situation in which something happens (in its context); to decontextualize is to remove the context/situation in which something happens and to think about it more generally.*

Draw and justify conclusions: *to come to a final decision or judgment; an opinion or decision that is formed after a period of thought or research, and to be able to explain one's reasoning for that conclusion.*

Examine claims: *to look at (what someone says is true when some people may say it is not true) closely and carefully in order to learn more about it, to find problems, etc.*

Identify and explain correspondences: *to know and be able to say when there is (a direct relationship to or with something or between two things)*

Listen to, read, and understand the **perspectives** and approaches of others: *to be open to hearing an argument or statement that may be different from one's own.*

Make **conjectures**: *to offer an opinion or idea formed without proof or sufficient evidence.*

Respond to arguments of others: *to say or write something as an answer to a question or an argument from someone else.*

Use assumptions with careful thought: *to offer an idea or argument that is believed to be true or probably true but that is not known to be true.*

Use/provide **clear, explicit definitions**: *to explain the meaning of a word, phrase, etc., in a way that others can grasp that meaning.*

Use counterexamples: *to provide examples that show a different or opposite perspective than the one under discussion.*

All definitions taken from the [Merriam Webster Learner's Dictionary \(2020\)](#).